The Stirling Engine

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1 Introduction

The Stirling motor - a heat engine which converts heat into work - is the second oldest heat engine. It has many positive properties, e.g. it only needs a temperature difference to work, irrespective of whether the difference is achieved by solar heating or conventional fuel. This makes it very flexible and beneficial to the environment.

I will discuss the thermodynamic principles necessary for the understanding of the operating of the Stirling engines. Taking into account that we have built a γ-Stirling motor in this laboratory, I will focus mainly on this type.

2 History

On November 16^{th}, 1816, the 26 year old Scottish cleric *Robert Stirling* secured the patent no. 4081 for his heat engine. His motivation was to provide an alternative to the high pressure engines common at the time because many workers died of exploding boilers. In 1827 he built an improved version with his brother James who was an engineer. Towards the end of the 19^{th} century small versions were quite common in the homes of the upper class. It is highly notable that they constructed the engine eight years before Carnot published his remarkable work about the Carnot process. During the last few years the Stirling engine has become quite popular again. In 1990 Dr. Senft established a world record on low temperature differential Stirling engines, running on a temperature difference of just 0,5 Kelvin.

3 Applications and Advantages

Because of its nature to exclude the heating as an external part, the whole system is closed and contains no valves and is therefore low-maintenance, silent and non-vibrating. It does not need oil and as a result no refilling. The temperature difference can also be achieved by environmental friendly fuels or even by solar energy, which is used in solar power plants and block heat power plants. And the only theoretical limit to its efficiency is the thermodynamical limit. It has been designed for the use in submarines and ships, e.g. by the Swedish company Kockums [1]. By putting work into the machine, it can be and is used as a cooling device without the need for a cooling agent in cryogenics. One disadvantage is that the engine runs best continuously, thus making fast power changes impossible. Besides, the bad power to weight ratio of most types is a burden for mobile applications.

4 The function of the Stirling motor

In order to be able to discuss all features of the Stirling engine, I would like to give a short repetition about the Carnot process. The following sub-chapter *The Ideal Stirling Process* should give an overview about the mechanism in the Stirling engine, which results then in the real Stirling process.

4.1 A little bit of thermodynamics

In this sub-chapter I want to present some terms and equation that will be needed later.

Now I want to explain what is meant by work and by heat. Work is defined as an amount of energy obtained or needed to change a system relative to a force, e.g. the gravitational force where as heat is the amount of energy in a system resulting from its thermodynamical nature.

The *ideal gas equation*

$$pV = nRT \tag{1}$$

is valid for an ideal gas, where the atoms do not occupy any volume and they interact only through elastic collisions. p stands for the pressure, V for the volume the gas occupies, n stands for the number of moles, R is the molar gas constant and T is the temperature. From now on, we will arbitrarily discuss a gas with one mol, which allows us to set n equal to one.

The 1^{st} *law of thermodynamics* describes the conservation of energy:

$$dQ = dU - dW \tag{2}$$

where dQ represents the amount of heat transfered to the system, dU describes the change of the internal energy of the system resulting in a change of temperature and dW stands for the work that is done by the system.

There are several processes with different behaviour, if you keep one parameter of the system fixed. In an *isothermal process*, the temperature T of the gas remains constant, such that $dQ = dW$ and $pV = const.$ For the *isochor process*, the volume V is constant and $dQ = dU$, while $dU = dW$ and $TV^{\gamma-1} = const.$ has to be used for the *adiabatic process* (no heat transfer $dQ = 0$).

4.2 The Carnot process

It is fundamental to understand the Carnot process [2] when discussing heat engines. Introduced 1824 by *Nicolas Léonard Sadi Carnot* in his work *Réflexions sur la puissance motrice du feu et sur les machines propres à* *développer cette puissance*, it is a process which does not change the total entropy S and is therefore reversible. That is the reason, why in reality, one can only achieve approximations to the Carnot process.

The (periodically working) Carnot machine is attached to a heat-bath and a cold-sink and contains of a piston in a cylinder. The heat-bath and the cold-sink have an infinite capacity, meaning that they can absorb or emit any amount of heat without ever changing their temperature. The Carnot process contains four steps:

1. an isothermal expansion at high temperature T_h
2. an adiabatic expansion from T_h to a lower temperature T_c
3. an isothermal compression at low temperature T_c and
4. an adiabatic compression form T_c to high temperature T_h

During the first step, a heat amount Q_h is taken from the heat-bath and the corresponding work $W_1 = RT_h \ln(\frac{V_b}{V_a})$ is done by the engine.

In the adiabatic, second step, the work $W_2 = c_v(T_h - T_c)$ is done by the engine.

Then in the third step, the heat Q_c is transfered from the engine to the cold-sink and the work $W_3 = RT_c \ln(\frac{V_c}{V_d})$ is done on the engine.

During the fourth step, the work $W_4 = c_v(T_c - T_h)$ is done on the engine.

From the adiabatic process, we see that $\frac{V_b}{V_a} = \frac{V_c}{V_d}$, resulting in $W_1 = W_3$. It follows that $Q_h - Q_c = W$, where

$$W = R(T_h - T_c)\ln(\frac{V_b}{V_a}) \tag{3}$$

is the total work that is done by the engine during the four steps.

The degree of efficiency $\eta = \frac{W}{Q_1}$ can be written as

$$\eta = \frac{T_h - T_c}{T_h} \tag{4}$$

There can not be a machine with a higher degree of efficiency or otherwise we could build a perpetuum mobile. We can also see that it is impossible to build an engine with the efficiency of 1. That means it is impossible to convert all heat into work because there cannot be a cold-sink with zero degree.

The Carnot process is independent of the gas used inside the engine. Real engines built cannot even achieve this ideal efficiency, because they can not work reversible taking into account unavoidable heat leakage and friction.

4.3 The parts of the Stirling engine

Although the parts of a Stirling engine vary depending of which type they are, the basics remain the same. There is a volume (or two connected at the α-type) which contains a gas as working medium that gets periodically heated and cooled by an external heat-bath and a cold-sink. The heating and cooling is adjusted by the displacer piston (or compressor piston at the α-type), which moves the gas to the areas of the heating. This results in a change of pressure which drives the working piston. The working-fluid is not changed during the process. Another important part is the flywheel, because like at the Carnot process, not every step is running automatically, making the need to put some energy into the engine during some steps. The role of the regenerator [3] is to absorb (ideally all) the heat amount which the gas emits during the isochor, fourth step of the ideal Stirling process (see below) and to provide the heat during the second step. The regenerator can be e.g. a block of steel-wool and can replace the displacer piston in $\beta-$ and $\gamma-$types.

4.4 The ideal Stirling process

I will use the γ-type to describe the process with respect to the fact that we built this type. The ideal Stirling process contains also four steps [4] : two isothermal and two isochor.

1. Let the first step be a isothermal compression at the temperature T_c. For the compression, the work $W_c = -RT_c \ln(\frac{V_{max}}{V_{min}})$ has to be done on the engine by a flywheel.
2. While the volume is at its minimum V_{max}, the displacer is in a position such that the gas gets heated up from T_c to T_h and the pressure increases steadily. The internal energy increases by $\Delta U_1 = c_v(T_h - T_c)$ which comes from the heat flow Q_1 out of the heat-bath.
3. Now the working piston moves such that the volume enlarges from V_{min} to V_{max}. During this isothermal process at temperature T_h, the work $W_e = RT_h \ln(\frac{V_{max}}{V_{min}})$ is done by the engine.
4. In this step the displacer piston has moved such that the gas gets cooled from T_h to T_c, lowering the pressure again while the volume has its maximum V_{max}. The internal energy is lowered by $\Delta U_2 = c_v(T_c - T_h)$ resulting in a heat flow Q_2.

So the usable energy is equal to

$$W = W_e + W_c = R(T_e - T_c) \ln(\frac{V_{max}}{V_{min}}) \tag{5}$$

During the isochor processes we have seen an extra heat flow with $\Delta U_1 = \Delta U_2$. If it is possible to store this heat during step 4 and reuse it in step 2, one can improve the efficiency η of the engine such that it gets equal to the one obtained by the Carnot process. This can be done by a regenerator and is discussed in the section *Improvements*.

4.5 The real Stirling process

While discussing the ideal Stirling process, we assumed isothermal processes. But to ensure such a process, it has to go infinitesimally slowly to always ensure an equilibrium of temperature. When building an engine it is difficult to realise the isochor processes while at the same time having an evenly running system. So in every Stirling engine built, the different steps are not separated strictly but overlap each-other.

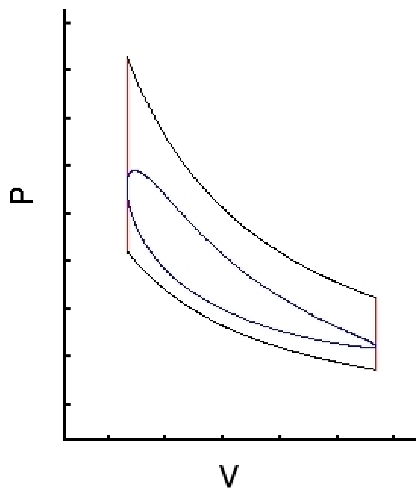

Figure 1: The real Stirling process in comparison with the ideal process

5 The different types

5.1 The α-type

This type [5] contains two cylinders which are normally arranged in an angle of 90 degrees. Because of this, it is also referred to as V-type. But there can be models found where the two pistons are coaxial. Normally one end is heated and the other end is cooled, but there are also version where the gas gets heated in the middle of the connecting piece of the two cylinders. It does not have a displacer piston, but a compressor piston. One way to realise this type is to heat next to the working piston and to cool the volume with the compressor piston. The connection part of the two cylinders can contain then the regenerator.

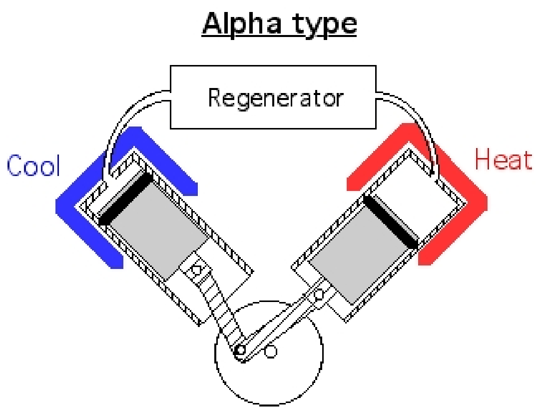

Figure 2: The α-type

These engines are used for medium to heavy loads because they can provide a better compression ratio $\frac{V_{max}}{V_{min}}$, compared with the other types, which allows a better power output. But it requires that two cylinders have to be sealed which results in more complex construction. For example, there are submarines and ships which use them.

5.2 The β-type

Stirling's patent drawing of the year 1816 shows a β-type [6]. It has only a single power piston and a displacer, which regulates if the gas gets heated up or cooled down. The piston and the displacer are located in the same volume which makes the driving mechanism more complicated.

5.3 The γ-type

Similar to the β-type, it has a working piston and a displacer piston, but they are not coaxial anymore, they are separately arranged in two different cylinders. Normally this results in more "dead" (or unswept) volumes than the other types. They are used if one wants to capitalise on the advantages of

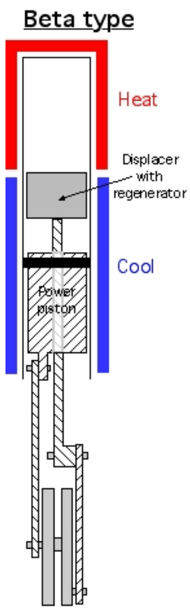

Figure 3: The β-type

having two cylinders. This type of engine is well suited for low temperature difference Stirling engines.

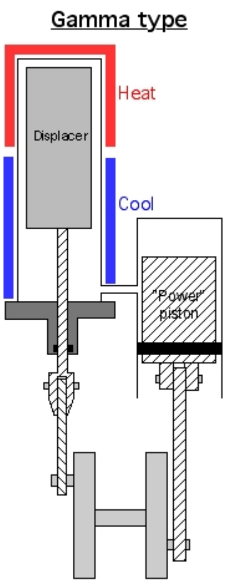

Figure 4: The γ-type

6 Possible improvements

There are several ways to improve our build engine. The simplest way is to heat at a higher temperature while cooling the other side. But this will only work up until a critical temperature because of the materials we have used. One has to ensure that there is as small as possible heat-leaking, which can be assured by using the right materials. Another possible improvement would be to replace the displacement piston by a regenerator normally with the disadvantage of a higher mass of the regenerator. The mass and the moment of inertia of the flywheel will influence the rotational frequency and can be adjusted to find an optimum. Also the phase shift of ninety degrees between the working piston and the displacer- (respectively the compression-) piston does not have to be the optimum and needs to be adapted to the machine. The use of a gas with a small heat-capacity also improves the efficiency of the engine because the gas can change the temperature quicker.

7 Conclusion

Its properties and features makes the Stirling engine a very interesting machine well worth to consider, but one cannot always overlook the disadvantages and limitations. By finding the appropriate field of application the Stirling engine has been and will be unbeaten for special tasks and it is up to the scientists to widen this field by pushing the limitations. This can be achieved by lowering the production costs, using lighter materials or by combining it with new inventions.

Even though the Stirling engine can be seen as dated, it is nowadays still perceived as useful and important. The increase in awareness of environmental issues and the need for more flexibility has made the Stirling engine even more relied upon due to its unique advantages in these areas.

A Notation

Symbol	Quantity
U	total energy of the system
Q	heat
W	work
S	entropy
T	temperature
η	efficiency
p	pressure
V	volume

References

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[3] http://peterfette.gmxhome.de/howdo.htm#A9

[4] http://de.wikipedia.org/wiki/Stirling_Motor#Funktionsweise

[5] http://www.ent.ohiou.edu/~urieli/stirling/engines/engines.html

[6] http://www.ent.ohiou.edu/~urieli/stirling/engines/beta.html

[7] http://www.ent.ohiou.edu/~urieli/stirling/engines/gamma.html